Feedback stabilization of the extrusion process modeled by a coupled hyperbolic system through a moving interface

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Control of PDE at CNAM

Outline

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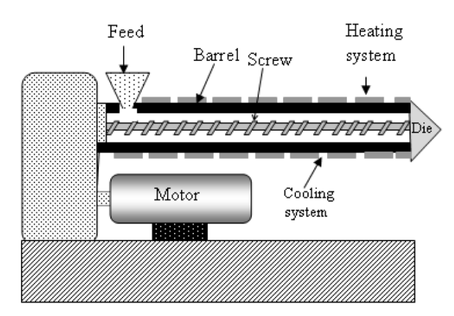
Extrusion process model

Extruders are designed to process highly viscous materials (polymer) in chemical industries and (mixture) in food industries. An extruder is made of a barrel with one or two Archimedean screws rotating inside, the temperature of which is regulated. The extruder is equipped with a die where the material comes out of the process.



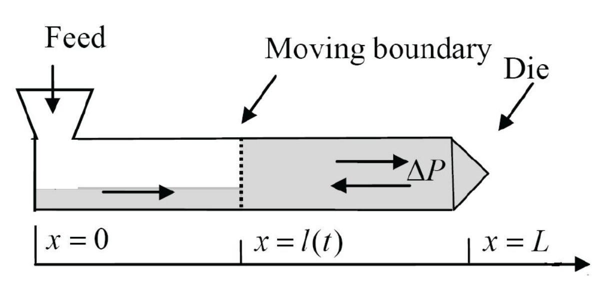
General features

- The extruder is of particular interest due to its modular geometry that allows the control of capacities of the mixtures along the machine.
- The filling ratio along the axial direction of the screws can be less than one in some part of the system according to the screw configuration and the operating conditions.
- The main phenomenon is obviously the fluid flow interacting with heat transfer and possibly chemical reactions.
- The fluid viscosity may significantly change due to the composition and temperature changes.
- The most important part of the extruder is the screw configuration which modulates extensively the mechanics.

Basic assumptions

- The flow is one-dimensional, laminar and strictly convective in the barrel direction.
- Heat losses are negligible along the axis of the screw.
- The specific heat capacity c_p and the density ρ_0 are assumed to be constant. The viscosity η is assumed to be constant.
- The screw pitch ζ is uniform along the extruder.
- There is no reaction which occurs along the process.
- The extruder is divided into time-varying spatial zones where the material fills completely or part of the extruder.
- There exists a thin interface $l(t)$ between the *PFZ* and the *FFZ* corresponding to the discontinuity of the filled ratio.
- The *PFZ* is submitted to the air pressure.

Mathematical modling



Parameters

B	Geometric parameter
c_p	Specific heat capacity
F_d	Net forward mass flow rate
K_d	Geometric parameter
S_{ech}	Exchange area between melt and barrel
V_{eff}	Effective volume
α	Heat exchange coefficient
S_{eff}	Effective area
η	Melt viscosity
μ	Viscous heat generation parameter
ρ_0	Melt density
ζ	Pitch length

The Partially Filled Zone (*PFZ*) $[0, T] \times [0, l(t)]$

f_p : Filling ratio, M_p : Moisture, T_p : Temperature of the mixture.

$$\partial_t \begin{pmatrix} f_p \\ M_p \\ T_p \end{pmatrix} = -\alpha_p(N)\partial_x \begin{pmatrix} f_p \\ M_p \\ T_p \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \Omega_p(f_p, N, T_p, T_{bp}) \end{pmatrix}$$

where

$$\alpha_p(N) = \zeta N$$

$$\Omega_p(f_p, N, T_p, T_{bp}) = \frac{\mu\eta N^2}{f_p \rho_0 V_{eff} c_p} + \frac{S_{ech}\alpha}{\rho_0 V_{eff} c_p}(T_{bp} - T_p)$$

$N(t)$: the rotation speed of the screw. $T_{bp}(t, x)$: the distributed barrel temperature.

The Fully Filled Zone (*FFZ*) $[0, T] \times [l(t), L]$

$f_f \equiv 1$. M_f : Moisture, T_f : Temperature. F_d : Net flow at the die.

$$\partial_t \begin{pmatrix} M_f \\ T_f \end{pmatrix} = -\alpha_f(N, l)\,\partial_x \begin{pmatrix} M_f \\ T_f \end{pmatrix} + \begin{pmatrix} 0 \\ \Omega_f(N, T_f, T_{bf}) \end{pmatrix}$$

where

$$\begin{aligned} \alpha_f(N, l) &= \frac{\zeta F_d(t)}{\rho_0 V_{eff}} \\ F_d(t) &= \frac{K_d}{\eta}(P(t, L) - P_0) \\ \Omega_f(N, T_f, T_{bf}) &= \frac{\mu\eta N^2}{\rho_0 V_{eff} c_p} + \frac{S_{ech}\alpha}{\rho_0 V_{eff} c_p}(T_{bf} - T_f) \end{aligned} \tag{1}$$

Relations at the Interface

Discontinuity of the filling ratio:

$$f_p(t, l(t)) < f_f(t, l(t)) = 1$$

Continuity of the temperature and moisture:

$$\big(M_p(t, l(t)), T_p(t, l(t))\big) = \big(M_f(t, l(t)), T_f(t, l(t))\big)$$

Pressure relation

$$\begin{aligned} \Delta P(t) &= P(t, L) - P_0 \\ &= \mathcal{P}(l(t), N(t)) := \frac{\rho_0 V_{eff} N(t) \eta (L - l(t))}{B\rho_0 + K_d (L - l(t))} \end{aligned} \quad (2)$$

obtained by integrating on $[l(t), L]$

$$\partial_x P(t, x) = \eta \, \frac{\rho_0 V_{eff} N(t) - F_d(t)}{B\rho_0} \quad (3)$$

Dynamics of the Interface

The interface dynamic is given by an ODE

$$\begin{cases} \dot{l}(t) = F\left(l(t), N(t), f_p(t, l(t))\right) \\ l(0) = l^0 \end{cases}$$

where

$$F\left(l(t), N(t), f_p(t, l(t))\right) = \frac{F_d(t) - \rho_0 V_{eff} N(t) f_p(t, l(t))}{\rho_0 S_{eff}(1 - f_p(t, l(t)))} \qquad (4)$$

Initial and boundary conditions

$$l(0) = l^0$$

$$(f_p(0,x), M_p(0,x), T_p(0,x)) = (f_p^0(x), M_p^0(x), T_p^0(x)),\ x \in (0, l^0)$$
$$(M_f(0,x), T_f(0,x)) = (M_f^0(x), T_f^0(x)),\ x \in (l^0, L)$$

$$\big(f_p(t,0), M_p(t,0), T_p(t,0)\big) = \Big(\frac{F_{in}(t)}{\rho_0 V_{eff} N(t)}, M_{in}(t), T_{in}(t)\Big)$$

where $F_{in}(t)$ is the feed flow rate.

Problems

- Well-posedness
  Open-loop system; Closed-loop system
  Existence, uniqueness, continuous dependence, regularity
- Stabilization
  Practical observers; Feedback law; Exponential stabilization
- Controllability
  State Controllability; Outflux controllability

Domain normalization for *PFZ*: $(0,T)\times(0,l(t)) \to (0,T)\times(0,1)$

$$\partial_t \begin{pmatrix} f_p(t,x) \\ M_p(t,x) \\ T_p(t,x) \end{pmatrix} + \alpha_p(t,x)\partial_x \begin{pmatrix} f_p(t,x) \\ M_p(t,x) \\ T_p(t,x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \Omega_p(f_p, N, T_p, T_{bp}) \end{pmatrix}$$

where

$$\begin{aligned} \alpha_p(t,x) &= \frac{1}{l(t)}\left(\zeta N(t) - x\dot{l}(t)\right) \\ &= \frac{1}{l(t)}\left(\zeta N(t) - xF(l(t), N(t), f_p(t,1))\right) \end{aligned}$$

$$\Omega_p(f_p, N, T_p, T_{bp}) = C_p(T_p - T_{bp}) + g_p(N, f_p)$$

$$C_p = -\frac{S_{ech}\alpha}{\rho_0 V_{eff} c_p}, \quad g_p = \frac{\mu\eta N^2(t)}{f_p(t,x)\rho_0 V_{eff} c_p}$$

Domain normalization for *FFZ*: $(0,T)\times(l(t),L)\to(0,T)\times(0,1)$

$$\partial_t \begin{pmatrix} M_f(t,x) \\ T_f(t,x) \end{pmatrix} + \alpha_f(t,x)\partial_x \begin{pmatrix} M_f(t,x) \\ T_f(t,x) \end{pmatrix} = \begin{pmatrix} 0 \\ \Omega_f(N, T_f, T_{bf}) \end{pmatrix}$$

where

$$\alpha_f(t,x) = \frac{1}{L-l(t)}\left(\frac{\zeta F_d(t)}{\rho_0 V_{eff}} + (x-1)\dot{l}(t)\right)$$
$$= \frac{1}{L-l(t)}\left(\frac{\zeta F_d(t)}{\rho_0 V_{eff}} + (x-1)F(l(t), N(t), f_p(t,1))\right)$$

$F_d(t) := F_d(l(t), N(t))$ is given by (1) and (2).

$$\Omega_f(N, T_f, T_{bf}) = C_p(T_f - T_{bf}) + g_f(N), \tag{5}$$

with

$$g_f := \frac{\mu\eta N^2(t)}{\rho_0 V_{eff} c_p}.$$

New initial and boundary conditions

New initial conditions:

$$(f_p(0,x), M_p(0,x), T_p(0,x)) = (f_p^0(x), M_p^0(x), T_p^0(x)),\ x \in (0,1)$$
$$(M_f(0,x), T_f(0,x)) = (M_f^0(x), T_f^0(x)),\ x \in (0,1)$$

New boundary conditions:

$$\big(f_p(t,0), M_p(t,0), T_p(t,0)\big) = \big(\frac{F_{in}(t)}{\rho_0 V_{eff} N(t)}, M_{in}(t), T_{in}(t)\big),$$
$$\big(M_f(t,0), T_f(t,0)\big) = \big(M_p(t,1), T_p(t,1)\big)$$

Norlmalized ODE-PDE coupled system

Nonlinear system:

$$\begin{cases} \dot{I}(t) = F\left(I(t), N(t), f_p(t,1)\right), \quad t \in (0,T), \\ I(0) = I^0. \end{cases} \tag{6}$$

$$\begin{cases} \partial_t f_p(t,x) + \alpha_p(t,x)\partial_x f_p(t,x) = 0, \quad (t,x) \in Q, \\ f_p(0,x) = f_p^0(x), \quad x \in (0,1), \\ f_p(t,0) = \dfrac{F_{in}(t)}{\rho_0 V_{eff} N(t)}, \quad t \in (0,T). \end{cases} \tag{7}$$

$Q := (0,T) \times (0,1)$

Norlmalized ODE-PDE coupled system

Linear system:

$$\begin{cases} \partial_t M_p(t,x) + \alpha_p(t,x)\partial_x M_p(t,x) = 0, \quad (t,x) \in Q, \\ \partial_t M_f(t,x) + \alpha_f(t,x)\partial_x M_f(t,x) = 0, \quad (t,x) \in Q, \\ M_p(0,x) = M_p^0(x), \quad M_f(0,x) = M_f^0(x), \quad x \in (0,1), \\ M_p(t,0) = M_{in}(t), \quad M_f(t,0) = M_p(t,1), \quad t \in (0,T). \end{cases} \tag{8}$$

$$\begin{cases} \partial_t T_p(t,x) + \alpha_p(t,x)\partial_x T_p(t,x) = \Omega_p(T_b), \quad (t,x) \in Q, \\ \partial_t T_f(t,x) + \alpha_f(t,x)\partial_x T_f(t,x) = \Omega_f(T_b), \quad (t,x) \in Q, \\ T_p(0,x) = T_p^0(x), \quad T_f(0,x) = T_f^0(x), \quad x \in (0,1), \\ T_p(t,0) = T_{in}(t), \quad T_f(t,0) = T_p(t,1), \quad t \in (0,T). \end{cases} \tag{9}$$

Well-posedness of (6)-(7)

Theorem (1)

Let (l_e, N_e, f_{pe}) *be s.t.* $F(l_e, N_e, f_{pe}) = 0$ *with* $0 < f_{pe} < 1$, $0 < l_e < L$. *Assume* C^0-*compatibility condition at* $(0,0)$. *Then,* $\exists\ \varepsilon_0$ *(depending on* T*) s.t. for any* $\varepsilon \in (0, \varepsilon_0]$, *if*

$$|l^0 - l_e| + \|f_p^0(\cdot) - f_{pe}\|_{W^{1,\infty}} + \|\frac{F_{in}(\cdot)}{\rho_0 V_{eff} N(\cdot)} - f_{pe}\|_{W^{1,\infty}} + \|N(\cdot) - N_e\|_{W^{1,\infty}} \leq \varepsilon$$

(6)-(7) $\exists!$ *solution* $(l, f_p) \in W^{1,\infty}(0,T) \times W^{1,\infty}(Q)$ *with*

$$\|l - l_e\|_{W^{1,\infty}} + \|f_p - f_{pe}\|_{W^{1,\infty}} \leqslant C_{\varepsilon_0} \cdot \varepsilon$$

Regularity of (6)-(9)

Theorem (2)

Assume further that $f_p^0(\cdot) \in H^2(0,1)$, $\dfrac{F_{in}(\cdot)}{\rho_0 V_{eff} N(\cdot)} \in H^2(0,T)$, *and the* C^1-*compatibility condition at* $(0,0)$ *holds. Then,* $\exists \varepsilon_0$ *(depending on T) s.t. for any* $\varepsilon \in (0, \varepsilon_0]$, *if*

$$|I^0 - I_e| + \|f_p^0(\cdot) - f_{pe}\|_{H^2} + \left\|\frac{F_{in}(\cdot)}{\rho_0 V_{eff} N(\cdot)} - f_{pe}\right\|_{H^2} + \|N(\cdot) - N_e\|_{H^2} \leq \varepsilon$$

(6)-(7) $\exists !$ *solution* $(I, f_p) \in W^{1,\infty}(0,T) \times C^0([0,T]; H^2(0,1))$ *with the estimate*

$$\|I(\cdot) - I_e\|_{W^{1,\infty}} + \|f_p - f_{p_e}\|_{C^0(H^2)} \leqslant C_{\varepsilon_0} \cdot \varepsilon$$

Remarks

Remark (1)

The solution we obtained in Theorem 1 or in Theorem 2 is called semi-global solution since it exists on any preassigned time interval $[0, T]$ if (I, f_p) has some kind of smallness (depending on T).

Remark (2)

We have the hidden regularity that $f_p \in C^0([0,1]; H^2(0,T))$ in Theorem 2.

Well-posedness of (8)-(9)

Theorem (3)

Let $T > 0$ *and* $M_p^0, M_f^0, T_p^0, T_f^0 \in L^2(0,1), M_{in}, T_{in} \in L^2(0,T)$ *be given. Then* (8)-(9) $\exists!$ *a unique solution* $(M_p, M_f, T_p, T_f) \in (C^0([0,T]; L^2(0,1)))^4$ *with*

$$\|M_p\|_{C^0([0,T];L^2(0,1))} \leqslant C \cdot \left(\|M_p^0\|_{L^2} + \|M_{in}\|_{L^2}\right),$$
$$\|T_p\|_{C^0([0,T];L^2(0,1))} \leqslant C \cdot \left(\|T_p^0\|_{L^2} + \|T_{in}\|_{L^2} + \|g_p\|_{L^2}\right),$$
$$\|M_f\|_{C^0([0,T];L^2(0,1))} \leqslant C \cdot \left(\|M_p^0\|_{L^2} + \|M_{in}\|_{L^2} + \|M_f^0\|_{L^2}\right),$$
$$\|T_f\|_{C^0([0,T];L^2(0,1))} \leqslant C \cdot \left(\|T_p^0\|_{L^2} + \|T_{in}\|_{L^2} + \|T_f^0\|_{L^2} + \|g_f\|_{L^2}\right)$$

Idea to prove Theorem (1)-(3)

Proof of Theorem (1)

Fixed point argument

$(I(\cdot), f_p(\cdot,1)) \mapsto \mathcal{F}((I(\cdot), f_p(\cdot,1))) := (\widetilde{I}(t), \widetilde{f_p}(t,1))$

Local existence + a priori estimates

Proof of Theorem (2) and (3)

Apply classical result on L^2 well-posedness of general linear transport equation

$$\begin{cases} u_t + a(t,x)u_x = b(t,x)u + c(t,x), \quad (t,x) \in Q, \\ u(0,x) = u_0(x), \quad x \in (0,1), \\ u(t,0) = h(t), \quad t \in (0,T), \end{cases}$$

Practical measurements and feedbacks

Indirect measurements: $P(t, L)$ or equivalently $\Delta P(t) := P(t, L) - P_0$.

Feedbacks: $N(t) = \mathcal{N}(\Delta P(t))$; $F_{in}(t) = \mathcal{F}(\Delta P(t))$ or

$$N(t) - N_e = k_1(\Delta P(t) - \Delta P_e) \quad (10)$$

$$F_{in}(t) - F_{ine} = k_2(\Delta P(t) - \Delta P_e) \quad (11)$$

$?\exists\ k_1, k_2 \in \mathbb{R}$ s. t. $(I, f_p) \to (I_e, f_{pe})$ as $t \to \infty$

Closed-loop system for (l, f_p)

$$\begin{cases} \dot{l}(t) = F\left(l(t), N(t), f_p(t,1)\right), \quad t \in (0,\infty), \\ \partial_t f_p(t,x) + \alpha_p(t,x)\partial_x f_p(t,x) = 0, \quad (t,x) \in (0,\infty)\times(0,1), \\ f_p(t,0) = \dfrac{F_{in}(t)}{\rho_0 V_{eff} N(t)}, \quad t \in (0,\infty). \end{cases} \tag{12}$$

where

$$\alpha_p(t,x) = \frac{1}{l(t)}\left(\zeta N(t) - xF(l(t), N(t), f_p(t,1))\right)$$

$$F\left(l(t), N(t), f_p(t, l(t)\right) = \frac{\frac{K_d}{\eta}\Delta P(t) - \rho_0 V_{eff} N(t) f_p(t, l(t))}{\rho_0 S_{eff}(1 - f_p(t, l(t)))}$$

$$\Delta P(t) := \mathcal{P}(l(t), N(t)) := \frac{\rho_0 V_{eff} N(t)\eta(L - l(t))}{B\rho_0 + K_d(L - l(t))} \tag{13}$$

Stabilization for (I, f_p)

Theorem (4)

Let $a_1 = \frac{\partial F}{\partial I}(I_e, N_e, f_{pe})$, $a_2 = \frac{\partial F}{\partial N}(I_e, N_e, f_{pe})$, $a_3 = \frac{\partial F}{\partial f_p}(I_e, N_e, f_{p_e})$, $b_1 = \frac{\partial \mathcal{P}}{\partial I}(I_e, N_e)$, $b_2 = \frac{\partial \mathcal{P}}{\partial N}(I_e, N_e)$. *If* $\exists (k_1, k_2) \in \mathbb{R}^2$ *s.t.*

$$a_1 + \frac{k_1 a_2 b_1}{1 - k_1 b_2} < 0 \tag{14}$$

$$\left| \frac{a_3 b_1 (k_2 - f_{p_e} \rho_0 V_{eff} k_1)}{\rho_0 V_{eff} N_e (1 - k_1 b_2)} \right| < \left| a_1 + \frac{k_1 a_2 b_1}{1 - k_1 b_2} \right| \tag{15}$$

then system (12) *under* (10)-(11) *is locally exponentially stable, i.e.,* $\exists\, \varepsilon > 0$, $M > 0$ *and* $\omega > 0$ *s.t.*

$$\text{If} \quad |I^0 - I_e|^2 + \|f_p^0(\cdot) - f_{p_e}\|^2_{H^2(0,1)} \leqslant \varepsilon \text{ with C.C.,} \quad \text{then} \tag{16}$$

$$|\bar{I}(t)|^2 + \|\bar{f}_p(t,\cdot)\|^2_{H^2(0,1)} \leqslant M e^{-\omega t} (|I^0 - I_e|^2 + \|f_p^0(\cdot) - f_{p_e}\|^2_{H^2(0,1)})$$

IFF condition for (14)-(15)

Proposition (1)

There $\nexists\ (k_1, k_2) \in \mathbb{R}^2$ *s.t.* (14)-(15) *hold if and only if*

$$\begin{cases} a_1 \geq 0 \\ a_2 b_1 = 0 \end{cases}$$

Practical parameters $\Rightarrow a_1 < 0$

Idea to prove Theorem (4)

- Linearization and perturbation
- Lyapunov function approach

$$L(t) = A_3(A_2(A_1 V_1(t) + V_0(t)) + V_2(t)) + V_3(t)$$
$$V_0(t) = \bar{I}^2(t)$$
$$V_1(t) = \int_0^1 e^{-\gamma_1 x} \bar{f}_p^2(t,x)dx$$
$$V_2(t) = \int_0^1 e^{-\gamma_2 x} f_{px}^2(t,x)dx$$
$$V_3(t) = \int_0^1 e^{-\gamma_3 x} f_{pxx}^2(t,x)dx$$
$$L(t) \simeq |\bar{I}(t)|^2 + \|\bar{f}_p(t,\cdot)\|^2_{H^2(0,1)}$$

Sketch of proof

Let k_1, k_2 be such that (14)-(15) are true.
For all smooth solution with $|\bar{I}(t), \bar{N}(t), \bar{f}_p(t,1)|$ small

$$\dot{V}_0(t) + A_1 \dot{V}_1(t) \leq -(\beta_0 + o(1)) V_0(t) - (\beta_1 + o(1)) V_1(t) - (\delta_1 + o(1)) \bar{f}_p^2(t,1)$$

$$\dot{V}_2(t) \leq -(\beta_2 + o(1)) V_2(t) - (\delta_2 + o(1)) f_{p_x}^2(t,1) + \theta_2 (V_0(t) + \bar{f}_p^2(t,1))$$

$$\dot{V}_3(t) \leq -(\beta_3 + o(1)) V_3(t) - (\delta_3 + o(1)) f_{p_{xx}}^2(t,1) + \theta_3 (V_0(t) + \bar{f}_p^2(t,1) + f_{p_x}^2(t,1))$$

Sketch of proof

$\exists A_1, A_2, A_3, \omega_0 > 0$ such that

$$\dot{L}(t) \leq -(\omega_0 + o(1))L(t)$$

If

$$|\bar{I}(t), \bar{N}(t), \bar{f}_p(t,1)| \leq \varepsilon_0. \quad (17)$$

then

$$\dot{L}(t) \leq -\omega L(t)$$

thus

$$L(t) \leq e^{-\omega t} L(0)$$

Therefore

$$|\bar{I}(t)|^2 + \|\bar{f}_p(t,\cdot)\|^2_{H^2(0,1)} \leq M e^{-\omega t}(|I^0 - I_e|^2 + \|f_p^0(\cdot) - f_{p_e}\|^2_{H^2(0,1)}) \leq M\varepsilon \quad \Rightarrow (17)$$

Stabilization for (M_p, M_f)

Measurements: $M_f(t,1)$

Feedbacks: $M_{in}(t) = \mathcal{M}(M_f(t,1))$ or

$$M_{in}(t) - M_e = k_3(M_f(t,1) - M_e)$$

Linear closed-loop system with known (I, f_p)

$$\begin{cases} \partial_t M_p(t,x) + \alpha_p(t,x)\partial_x M_p(t,x) = 0, \quad (t,x) \in Q, \\ \partial_t M_f(t,x) + \alpha_f(t,x)\partial_x M_f(t,x) = 0, \quad (t,x) \in Q, \\ M_p(0,x) = M_p^0(x), \quad M_f(0,x) = M_f^0(x), \quad x \in (0,1), \\ M_p(t,0) = M_{in}(t), \quad M_f(t,0) = M_p(t,1), \quad t \in (0,T). \end{cases}$$

$\exists\, k_3 \in \mathbb{R}$ s. t. $(M_p, M_f) \to (M_e, M_e) \in (L^2(0,1))^2$ as $t \to \infty$

Stabilization for (T_p, T_f)

Measurements: $T_f(t,1)$
Feedbacks:

$$T_{bp}(t,x) = \mathcal{T}_1(I(t), f_p(t,x))$$
$$T_{bf}(t,x) = \mathcal{T}_2(I(t), f_p(t,x))$$
$$T_{in}(t) - T_e = k_4(T_f(t,1) - T_e)$$

Linear closed-loop system with known (I, f_p)

$$\begin{cases} \partial_t T_p(t,x) + \alpha_p(t,x)\partial_x T_p(t,x) = \Omega_p(T_{bp}(t,x)), & (t,x) \in Q, \\ \partial_t T_f(t,x) + \alpha_f(t,x)\partial_x T_f(t,x) = \Omega_f(T_{bf}(t,x)), & (t,x) \in Q, \\ T_p(0,x) = T_p^0(x), \quad T_f(0,x) = T_f^0(x), & x \in (0,1), \\ T_p(t,0) = T_{in}(t), \quad T_f(t,0) = T_p(t,1), & t \in (0,T). \end{cases}$$

$\exists\, \mathcal{T}_1, \mathcal{T}_2$ and k_4 s. t. $(T_p, T_f) \to (T_e, T_e) \in (L^2(0,1))^2$ as $t \to \infty$

Lyapunov functions

Lyapunov functions:

$$V_4(t) := \int_0^1 e^{-\gamma_4 x} \bar{M}_p^2(t,x)\,dx + A_4 \int_0^1 e^{-\gamma_5 x} \bar{M}_f^2(t,x)\,dx$$

$$V_5(t) := \int_0^1 e^{-\gamma_6 x} \bar{T}_p^2(t,x)\,dx + A_5 \int_0^1 e^{-\gamma_7 x} \bar{T}_f^2(t,x)\,dx$$

$\exists\ \gamma_4, \gamma_5, \gamma_6, \gamma_7, A_4, A_5, k_3, k_4, \omega_4, \omega_5$ such that

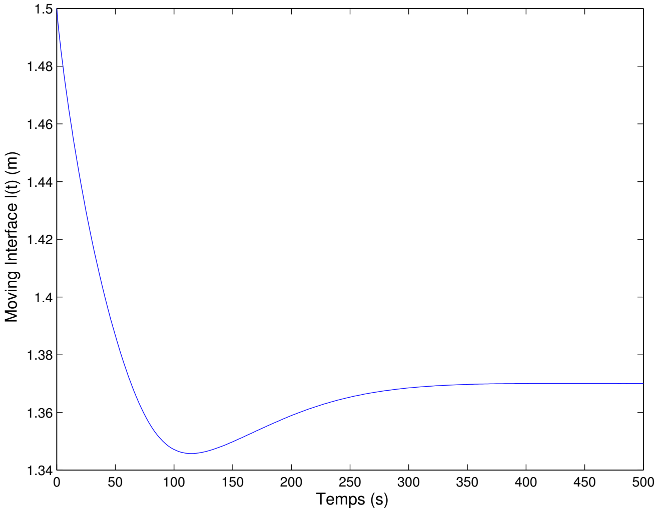
$$\dot{V}_4(t) \le -\omega_4 V_4(t)$$

$$\dot{V}_5(t) \le -\omega_5 V_5(t)$$

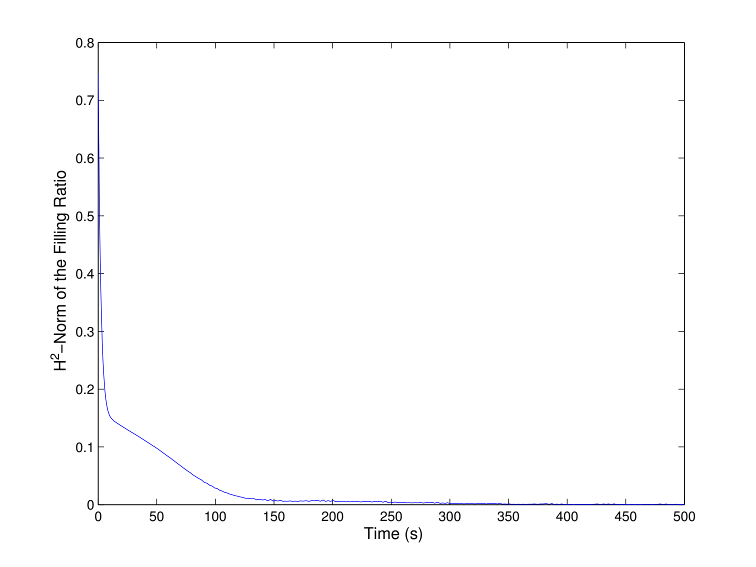
Numeric simulations

- A semi discretization scheme based on the finite volume method is applied for the simulation of the transport equation of the system with the control law. Time integration is performed with ODE45 routine of MATLAB.
- Data
  Equilibrium: f_{pe}=0.6 $N_e = 250rpm$ $l_e = 1.43m$
  $F_{ine} = 131.9471kg/h$ $\Delta P_e = 2.2907e+03Pa$
  $M_e = 0.25$ $T_e = 380K$
  Initial data:
  $f_p^0(x) = 0.6908 + 0.025(1 - cos(\pi x)) + 0.6149sin(\pi x)/\pi$
  $l^0 = 1.5m$ $M_0 = 0.1$ $T_0 = 293K$
  Feedback Gain: $k_1 = 0.1$ $k_2 = 0.0001$ $k_3 = -0.05$
  $k_4 = -0.05$

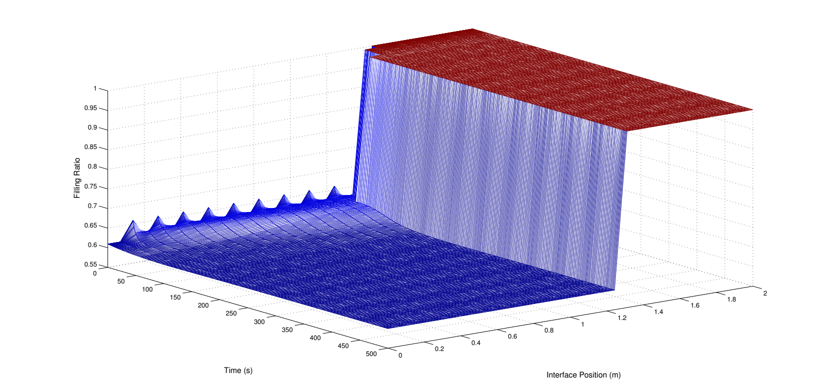
Simulation for the interface



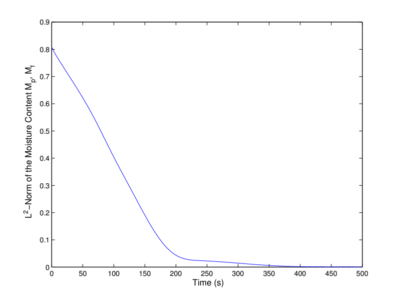
Stability of the filling ratio

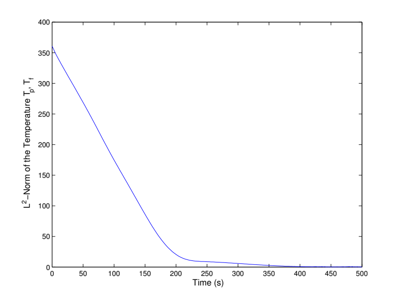


Simulations for the interface and the filling ratio



Stability for moisture and temperature





Conclusion

- Well-posedness of a physical model for the extrusion process, described by two systems of conservation laws coupled with a dynamical interface.

- Exponential stabilization is obtained for the closed-loop system with natural feedback controls.
  Lyapunov approach
  Numerical simulations

- Main difficulty:
  PDE-ODE nonlinear coupling
  Free boundary problem
  Feedbacks with indirect measurements

Perspectives

- Rapid Stabilization
  Fast decay rate
- Controllability
  State controllability; Outflux controllability
- Distributed viscosity case
  $\eta(t,x) = \eta(M_f(t,x), T_f(t,x))$ in (3)
- Heat diffusion

Perspectives

Merci!